By the Numbers: An exploration of Zipf's Law: does it apply to Canada and Ontario?

An exploratory brief report

David I.M. Clark MA, BA(Honours), BES, MAd(Diploma)
DMCmetrix.com

An exploration of Zipf's Law: does it apply to Canada and Ontario?

David I.M. Clark. MA, BA(Honours), BES, MAd(Diploma) DMCmetrix

A phenomenon which is known as the "rank-size rule" was popularised by G.K. Zipf in 1949.[i] What the rank-size rule states, as applied to population and cities, is that when cities are ranked from highest to lowest in population and graphed against its rank, the result is a very straight line, typically with a slope of -1.0. And further, that the second largest population (rank 2) is about one-half the population of the largest city; the third-largest is about one-third the size of the largest; etcetera (a power law). For USA cities, this has held to be true since at least 1890.[ii]

This phenomenon also applies to frequency of English (and other) language words, biology, meteorite strikes, and earthquakes.[iii]

This research brief explores whether or not Zipf's Law applies to cities in Canada, and we'll explore Ontario's urban communities and its counties as to application of the law.

Figure 1 shows the plot for 208 of the largest urban population centres in Canada in 2001, showing a strong negative slope, with data points forming almost a straight line. [iv] The slope is - 0.9945. There are several anomalies. The largest cities plot as "primate distributions" which is typical of key cities such as political capitals[v] and financially-important centres. In fact, the circled cities are Toronto, Montreal, Vancouver, Calgary, Edmonton, Ottawa, Quebec City, Winnipeg, and Hamilton, mostly capital cities.

Figure 2 is updated to 2011. The slope is -0.9984, again very close to a straight line. Cities number four to nine still show those primate positions.

I tested Zipf's Law with population figures for the 49 counties of Ontario (Canada), and towns within Grey and Bruce counties. The resultant slopes do not reflect an adherence to Zipf's Law (see Table 1). For Grey and Bruce, this failure to adhere might reflect the small number of communities (n=17, mostly small towns and rural), and a lack of spatial density of towns and cities. The counties' lack of adherence might reflect that fact that these political units are geographically large and lack the spatial density of cities.

I further tested Zipf's Law with one segment of the Grey and Bruce counties' population, specifically seniors (aged 65 years and older). This is due to an interest which I have